Confidence Intervals

Confidence Interval for a Population Mean: $\sigma$ Known

**Example 1** Find a 95% confidence interval for the starting salaries of college graduates who have taken a statistics course where $n = 28$, $\bar{x} = $45,678, $\sigma = $9,900, and the population is normally distributed.

1. Press STAT
2. Arrow to TESTS
3. Select 7:ZInterval...
4. Highlight Stats
5. Press $\downarrow$ and enter the values for $\sigma$, $\bar{x}$, $n$, and C-Level.
6. Highlight Calculate, then press ENTER.

We are 95% confident that the mean starting salary of college graduates that have taken a statistics course is between $42,011 and $49,345. This means that if we were to
select many different samples of size 28 and construct 95% confidence intervals for each sample, 95% of the constructed confidence intervals would contain \( \mu \) and 5% would not contain \( \mu \). It is incorrect to say “there is a 95% chance that \( \mu \) will fall between $42,011 and $49,345.” The population mean, \( \mu \), is not a random variable, it is a fixed, but unknown, constant. The probability that this interval contains \( \mu \) is 0 or 1.

Confidence Interval for a Population Mean, \( \sigma \) Unknown

Example 2 Find a 95% confidence interval for the starting salaries of college graduates who have taken a statistics course where \( n = 28 \), \( \bar{x} = $45,678 \), \( s = $9,900 \), and the population is normally distributed.

1. Press STAT
2. Arrow to TESTS
3. Select 8:TInterval...

4. Highlight Stats
5. Press \( \downarrow \) and enter the values for \( \bar{x} \), \( s \), \( n \), and C-Level.

6. Highlight Calculate, then press ENTER.
We are 95% confident that the mean starting salary of college graduates that have taken a statistics course is between $42,011 and $49,345.

**Note:** The confidence interval using the \( t \) statistic is wider than the interval using the \( z \) statistic, even though the sample sizes are the same and the same value for \( \sigma \) and \( s \) is used. The reason for this is that the primary difference between the sampling distribution of \( t \) and \( z \) is that the \( t \) statistic is more variable than the \( z \), which seems obvious when you consider that \( t \) contains two random quantities (\( \bar{x} \) and \( s \)), whereas \( z \) contains only one (\( \bar{x} \)). Thus, the \( t \) value will always be larger than a \( z \) value for the same sample size.

**Example 3** The following random sample was selected from a normal distribution: 4, 6, 3, 5, 9, 3. Construct a 95% confidence interval for the population mean, \( \mu \).

1. Enter the data into \( L1 \).
2. Press **STAT**.
3. Arrow to **TESTS**.
4. Select **8:TInterval...**
5. Highlight **Data**.
6. Press **\( \downarrow \)** and enter the values for **List**, **Freq**, and **C-Level**.
7. Highlight **Calculate**, then press **ENTER**.
We are 95% confident that the population mean, \( \mu \), is between 2.6 and 7.4.

**Confidence Interval for a Population Proportion**

**Example 4** Public opinion polls are conducted regularly to estimate the fraction of U.S. citizens who trust the president. Suppose 1,000 people are randomly chosen and 637 answer that they trust the president. Compute a 95% confidence interval for the population proportion of all U.S. citizens who trust the president.

1. Press **STAT**
2. Arrow to **TESTS**
3. Select **A:1-PropZInt...**

4. Enter the values for \( x \), \( n \), and **C-Level**.

```
1-PropZInt
x: 637
n: 1000
C-Level:.95
Calculate
```

5. Highlight **Calculate**, then press **ENTER**.
We are 95% confident that the true percentage of all U.S. citizens who trust the president is between 60.7% and 66.7%.