The Vision, the Challenges, and the Reality: Improving the Undergraduate Experience of Teachers

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Introduction

Teachers’ knowledge of mathematics includes many and varied ways of representing and presenting different parts of mathematics. This includes knowledge of clarifying examples, analogies, and connections between ideas. Teachers also, vitally, need to address students’ learning needs and strive to enhance student learning (Grouws and Schultz, 1996; NRC, 2001, p. 189). To do this, knowledge of how mathematics can be taught (i.e., pedagogical content knowledge) is requisite. Examples of the pedagogical content knowledge needed by mathematics teachers is provided by the CBMS Mathematical Education of Teachers document, the NCTM Principles and Standards for School Mathematics 2000, and in the books of Liping Ma (1998), Stigler & Hiebert (1999), and Bransford, Brown & Cocking (1999).

Acknowledgement of the need to address the special nature of mathematics knowledge needed by teachers has profound implications for those who teach mathematics courses at institutions of higher learning. We need a careful rethinking of what constitutes an appropriate and useful preparation for prospective teachers in light of changing expectations and standards. Bodies such as the National Research Council (2000, 1999, 1996) and the National Science Foundation (1998; 1996) have identified and examined the issues involved in improving mathematics teacher education. The MET recommendations present a vision of mathematics instruction for future teachers that is exciting to think about and daunting in its challenges. This report recommends that:

• mathematics courses develop an in-depth understanding of the mathematics future teachers will be expected to teach.
• courses about school mathematics should focus on a thorough development of basic mathematics ideas.
• all courses designed for future teachers should develop careful reasoning and mathematical “common sense” in analyzing conceptual relationships and in applied problem solving.
• prospective teachers should be exposed to flexible, interactive teaching, especially in
courses on school mathematics and should understand that there are multiple ways to
engage students in mathematics.

In this paper, I focus on some of the practical issues of implementation which are raised by
these recommendations—issues which need to be addressed in open forums by the various stake-
holders in the mathematics preparation of teachers. Reflection on these MET recommendations
generates more questions than answers: What constitutes “in-depth” understanding of mathemat-
ics at various levels of the curriculum? Which “basic” ideas should content courses about school
mathematics focus on? What understandings about notions of proof and justification are reason-
able for prospective elementary teachers, for secondary teachers?

The many challenges to a successful implementation of the MET recommendations, include
the following:

• The establishment of essential, long-term partnerships among mathematicians, mathe-
  matics educators, mathematics education researchers, and the various other constitu-
  encies with interest in improving teacher preparation.

• Development of a shared understanding of what is meant by “a deep understanding of
  school mathematics” and how to assess it.

• Recognition by faculty who teach the general education mathematics courses of the
  first two years, whether at two-year and four-year colleges or at universities, that they
  are involved in the preparation of future teachers.

• Objective examination of faculty belief structures and values which determine the
  nature of the classroom environment that shapes students’ beliefs about the nature of
  mathematics.

• Acceptance and adoption of systemic change models of improvement.

• A resolution of issues of articulation which are roadblocks to the formation of a
  coherent vision of mathematics education.

The role of the two-year college in teacher preparation

Various reports urge the establishment of essential partnerships and collaborative efforts
among the various groups and institutions involved in the mathematics education of prospective
teachers. Each of these reports specifically acknowledges that two-year colleges are essential part-
ners in this endeavor. The MET recommendations state that:

A two-year college’s students transfer to a variety of different four-year institutions
with differing course requirements in mathematics (and other sub-jects) for future
teachers. Good articulation agreements are needed with mathematics departments
and education schools at nearby baccalaureate institutions about the coursework for
future teachers.
According to a recent NSF report, *Investing in Tomorrow’s Teachers* (1999), an estimated 40% of teachers complete some or all of their science and mathematics coursework at two-year colleges (NSF, 1999, p. 6). Many future elementary and middle school teachers take most, if not all, of their mathematics content courses at two-year colleges. Two-year colleges accounted for 46% of all collegiate mathematics enrollment in Fall, 1995. The CBMS Fall 1995 Survey predicted that, “By the turn of the century, mathematics enrollment in two-year colleges will equal or exceed enrollment in four-year colleges and universities” (CBMS Fall, 1995 Survey, p. 4).

As the costs of tuition and room and board continue to increase, and as more students need to work to pay the costs of their schooling, the local community college is increasingly the choice of many students. Mathematics courses that show big percentage increases since the CBMS Fall, 1990 survey include developmental algebra, college algebra, precalculus, elementary statistics, and the mathematics content courses for preservice elementary teachers. In 1995, the Math for Elementary Teachers course was offered at 43% of the two-year colleges with Mathematics programs (CBMS Fall 1995 Survey, p. 6).

Additionally, mathematics courses at two-year colleges enroll many students from the general college population who are unidentified prospective secondary teachers. The National Research Council recently published a report, *Educating Teachers of Science, Mathematics, and Technology: New Practices for the New Millennium* (NCR, 2001). In that report, the Committee on Science and Mathematics Teacher Preparation reminds higher education communities in science, mathematics, engineering, and technology that:

...all colleges and universities, including those which do not have formal teacher education programs, should become more involved with improving teacher education because the nation’s teacher workforce consists of many individuals who have matriculated at all types of two- and four-year institutions of higher education. Although many of these schools do not offer formal teacher education programs, virtually every institution of higher education, through the kinds of courses it offers, the teaching it models, and the advising it provides to students, has the potential to influence whether or not its graduates will pursue careers in teaching.... Science, mathematics, and engineering departments at two- and four-year colleges and universities should assume greater responsibility for offering college-level courses that provide teachers with strong exposure to appropriate content and that model the kinds of pedagogical approaches appropriate for teaching that content. (NRC, p. 9-12).

a New Millennium (2001), together with the MET recommendations provide a framework and direction for the expansion of collaborative efforts between two- year and four-year colleges engaged in the undergraduate preparation of teachers. Mathematics departments at two-year and four-year colleges and universities are challenged to:

- assume greater responsibility for offering college-level course that provide teachers with strong exposure to appropriate content and that model the kinds of pedagogical approaches appropriate for teaching that content.
- re-examine and redesign introductory college-level courses in mathematics to better accommodate the defined needs of practicing and future teachers.
- maintain contact with and provide guidance to teachers who complete their preparation and development programs.
- assume primary responsibility for providing professional development opportunities to experienced teachers of mathematics, following a period of collaborative planning and preparation.

Articulation and alignment of expectations across institutions

The growing national recognition that the community college is an essential partner in the mathematical preparation of prospective teachers is an acknowledgment of the reality of how many students begin their coursework at community colleges. Two-year and four-year institutions in close proximity are demonstrating that it is possible to work together to jointly develop a comprehensive, coherent teacher preparation program that builds on the foundation provided at the two-year institution. Models of collaborative programs do exist and the number of such collaborations is growing. Collaborative initiatives to improve the undergraduate preparation of future teachers include Henry Ford Community College and University of Michigan-Dearborn; Green River Community College in Auburn, Washington and Central Washington University; Cerritos College in Norwalk, California and California State University-Long Beach; and the Virginia Community College System (23 community colleges). In Illinois, a NSF–funded collaborative has brought together mathematics and science faculty from seven institutions: University of Illinois-Chicago, three city colleges: Truman, Olive Harvey and Harold Washington; and three suburban community colleges: William Rainey Harper, Oakton, and Triton. Goals of this collaborative include efforts to improve the teaching and learning of mathematics in the first two years of college, implementation of various teacher preparation initiatives including recruitment, mentoring of first and second year practitioners, and improved articulation of the mathematics content courses and general education science courses.
The issue of articulation, however, and the alignment of expectations and assessments across institutions are on-going concerns for faculty involved in teacher preparation, not only at the two-year colleges, but also at many transfer institutions. Many states, regions, or local higher education alliances have developed articulation agreements which allow students to transfer in with junior status when they have completed a specified “core curriculum” or a two-year A.A. or A.S. degree. However, the reality is that, even with articulation agreements in place, students continue to experience difficulties with transfer.

A smooth articulation from two-year to four-year programs of teacher preparation is problematic for many students because of the lack of a shared vision and clear expectations of how students demonstrate growth of a “deep understanding of school mathematics.” Mathematics content courses in elementary teacher preparation programs at many four-year institutions are evolving at the same time two-year programs are restructuring their general education courses to meet agreed-to articulated guidelines. Some transfer institutions have not yet figured out how two-year college mathematics content courses fit into their evolving programs. A placement process which assesses the conceptual understanding of students as well as their skills competency upon entrance to programs currently being revised at many transfer institutions would be a fine thing. It has yet to be determined.

Information and conversations with both two-year and four-year colleagues involved in teacher preparation at various national and regional conferences each year indicate that the mathematics courses taken during the first two years, at both two-year and four-year institutions, results in widely divergent experiences which contribute to the articulation difficulties cited in various reports and by the MAA Task Force on Articulation.

The principle cause of the transition problems in US mathematics education has been described as the lack of an intellectually coherent vision of mathematics among the professionals responsible for mathematics education. The sometimes heated and often public disagreements about the nature of mathematics as well as effective ways to teach it have led to a bewildering variety of curricular and pedagogical approaches (Focus, January, 2001, p. 10).

There are states which do not offer any mathematics content courses designed specifically for prospective elementary teachers in either their two-year colleges or in their four-year programs. Students in these states take a traditional College Algebra course as their preparation for teaching elementary grades mathematics K-8. This can mean anything from Beginning Algebra to a Col-
lege Algebra course that includes theory of equations. Some institutions offer a single mathematics content course. Still other two-year schools and many four-year college and university elementary teacher preparation programs offer two content courses.

The number of credit hours granted for mathematics content courses for preservice elementary teachers varies from 3 semester hours to 5 semester hours per course. Some programs offer a single 4 credit-hour semester mathematics content course of fourteen weeks in which the enrollment is 100 or more students per section. Nearby two-year colleges offer the two-course general education sequence required by state articulation agreements. Each course is 4 credit-hours, often with five contact hours per week, over sixteen weeks, with classes that generally number twenty to twenty-five students per section.

Prerequisites for the mathematics content courses are also inconsistent, varying within a given state and nationally. In several states, the mathematics content course carries a prerequisite of Beginning Algebra. In other states, the prerequisite is Intermediate Algebra. Still other states and institutions with elementary teacher preparation programs require a course called College Algebra as the prerequisite course.

Given the diversity in content, number of hours, and number of courses offered at four-year and at two-year colleges, the question of how to effect a smooth articulation and continuity in the mathematical preparation of prospective teachers is one that urgently needs to be addressed by all those concerned with teacher preparation. These different curricular routes for students need to be considered and evaluated within the context of goals to which the various parties involved in the mathematics education of teachers can agree.

Mathematical knowledge, attitudes, beliefs, and values

A significant number of prospective K-8 teachers begin their college mathematics coursework at two-year colleges enrolled in developmental programs. They take one or more semesters of remedial algebra prior to taking the content course. The MET recommendations acknowledge the practical difficulties involved in teaching students who enter college with deficiencies in their mathematics background, a fear of mathematics, and poor learning skills. The more difficult task of changing firmly held beliefs and values about the nature of mathematics and how it is learned is not adequately discussed in the report.

How do prospective elementary teachers view mathematics as they begin the mathematics content course? Their mathematical autobiographies tell a repetitive story about their previous
mathematical experiences. and describe These descriptions of their experiences and increasing
difficulty in learning mathematics as they advance through the educational system are typical of
many pre-service elementary teachers’ experiences:

When I began learning mathematics everything was so simple! As I got older there
were many more rules being taught to me. The more rules I learned, the easier it
came to forget some of the older rules.

Coming into this class, I was under the impression that finding a formula to solve a
problem was, in reality, the answer to the problem. Formula finding, as the sole solu-
tion in mathematics, was an obstacle I quickly discovered I would need to overcome.
I have attempted to change my perception that math is not just about getting the right
answer.

This strictly utilitarian perspective of mathematics often limits students’ mathematical vision.
Were they to be anything other than teachers this restricted view of mathematics might not be so
important. For the education of their students, however, overcoming these limitations is critical.
We are concerned when preservice teachers figure the answer to $9 + \frac{35}{8}$ by adding $\frac{72}{8} + \frac{29}{8}$. We
are frustrated when, after building towers of height 4, using blocks of 2 different colors, they jus-
tify their result with the statement: “I know I have found all possible towers because other groups
in the classroom got the exact same answer.”

Changing students’ utilitarian view of mathematics and changing what they value in mathe-
matics is much more difficult than teaching them algorithms. Effecting lasting changes in those
beliefs and values present significant practical and theoretical challenges for mathematicians who
teach the content courses to prospective teachers. How do we change this severely procedural ori-
etnation to mathematics focused on ‘correct answers’ and peer validation?

Still other issues of implementation include (Schoenfeld, 1994, p. 357-358):

- the time required to catalyze such changes with sustained attention to both
cognitive and metacognitive processes.
- the task of creating the “right” instruction context, and providing the appropriate
kinds of modeling and guidance.
- the lack of a research base to inform practice.

How much time and consistent reinforcement throughout their academic careers is required to
effect these changes? What are the consequences of continuing to graduate students, intent on
becoming mathematics teachers, who remain inflexibly fixed in their utilitarian beliefs? The time
necessary to effectively change attitudes and beliefs of undergraduate prospective teachers; the resolution of these other implementation issues remain open and basic questions.

**The assessment of “deep understanding”**

What constitutes “deep understanding” of mathematics? What does this mean? Liping Ma (1999) has provided us with several examples of deep understanding of school mathematics. These are summarized in the recent *Educating Teachers of Science, Mathematics, and Technology: New Practices for the New Millennium* report:

1. The ability to sequence appropriately the introduction of new concepts;
2. The ability to make careful choices about problem types to be given to students in terms of number, context, and difficulty;
3. Brief, but significant opportunities for students to encounter conceptual obstacles;
4. Solicitation from and discussion by students of multiple points of view about a problem;
5. Anticipation of more complex and related structures;

Ma points out that a profound understanding of elementary mathematics is developed over years of teaching experience and on-going professional discussions with colleagues. If the development of profound understanding requires years of experience, this raises many important questions:

- What are the various stages in the development of profound understanding of mathematics?
- What level(s) of understanding are appropriate and realistic for prospective teachers?
- What content is appropriate for prospective teachers who will be using a variety of text materials, from very traditional to reform materials which place much greater demands on teachers’ mathematical knowledge?
- What does “deep understanding” look like in a multi-course program across institutions—in general mathematics courses of the first two years of college, in the mathematics content course(s) for preservice elementary teachers, in the mathematics major courses of secondary teachers, in the methods courses?
- How stable is the student’s learning?
- Will the growth of understanding continue in subsequent courses and when the student becomes a practitioner in the field?
- What should the teachers of teachers have as goals of instruction in order that they might design their instructional practices in ways which promote the building of rich, connected webs of mathematical concepts and skills?
How is “deep understanding of elementary mathematics” to be assessed? This difficult question has yet to be answered. Lynn Steen reminds us that “assessment should measure what is worth learning, not just what is easy to measure” (Gold, Keith, & Marion, 1999, p. 3). A shared consensus of the expectations of students with respect to the development of “deep understanding of school mathematics” in mathematics content and methods courses, of practitioners after one or two years in the field, and after several years in the field, is critical for successful implementation of the MET recommendations.

Faculty who teach these courses need to know how this understanding can be measured in ways that support every student’s opportunity to learn important mathematics. How will assessment practices be aligned with instructional goals that promote the growth of deep conceptual learning in a process that involves many constituencies? Matching faculty expectations and those of others with stakes in the outcomes requires an open assessment process and some consensus about what constitutes an authentic assessment of “understanding.” Do multiple measures of performance, including students’ self-evaluations, interview data, and portfolios when combined with objective tests provide authentic assessment of what students know at a given moment in time? How will retention of learning be measured? What criteria should be used to determine whether “satisfactory growth in understanding” is sufficient to meet articulation requirements and/or teacher certification requirements? In what forums will these questions be collaboratively and openly examined?

**Possibilities for implementing a process of improvement**

Different suggestions for on-going professional development of classroom teachers have been set forth by Stigler and Hiebert (2000) and by Ball and Cohen (1999). Stigler and Hiebert suggest that lesson study is “a process of improvement that is expected to produce small, incremental improvements in teaching over long periods of time.” They identify a number of interesting aspects of lesson study based on an analysis of Japanese practice that appear to be consistent with what is known about changing complex, cultural activities. Assuming that change occurs incrementally over time, they argue that lesson study:

- is based on a long-term continuous improvement model.
- maintains a constant focus on student learning.
- focuses on the direct improvement of teaching in context.
is collaborative and results in a shared language for describing and analyzing classroom teaching while providing a benchmarking process that teachers can use to gauge their own skills.

Stigler and Hiebert argue that lesson study encourages teachers to see themselves as contributors to the growth of knowledge about teaching as well as to their own professional development. Consequently, they argue, teachers become willing participants and agents of change.

Ball and Cohen (1999) suggest another possible approach for professional learning and ongoing development. It is their belief that “teachers must be actively learning as they teach” (p. 11). For them, the primary purpose of teacher education “is to cultivate the knowledge, skills, and values that will enable teachers to be highly effective in helping students learn.” (Ball & Cohen, 1999, p. 12). Ball and Cohen believe that the use of records of practice are important to change the discourse of practice among teachers and will result in improved teaching. They highlight the importance of teachers’ access to and participation in communities of practice.

Case studies and records of practice have been used effectively by experienced researchers. They provide “existence proofs” of effective teacher behaviors and classroom practices and are being used in professional development activities with K-8 classroom teachers. Examples of case studies and records of practice drawn from actual elementary classrooms are included in the MET document.

Are these improvement models appropriate for use in undergraduate mathematics content courses? To what extent will the use of these materials be effective with students whose beliefs about mathematics and values are already formed by years of experience? Prospective teachers, at the time they take their content courses, have had little, if any, actual teaching experience. To what extent are students able to critically examine records of practice when they have not yet developed a foundational base of mathematical experiences upon which to draw? Schoenfeld reminds us that “adapting these efforts for use in standard [K–8] classrooms by teachers who do not have the luxury to reflect on the issues and prepare or modify instruction requires a substantial amount of conceptualizing and pedagogical engineering” (Schoenfeld, 1992, p.358).

The role of mathematics faculty in teacher preparation

Unless current instructional practices and goals of those who teach undergraduate content courses change, improving the undergraduate experiences of prospective elementary and secondary mathematics teachers is problematic. Some argue that students abstract their beliefs about for-
mal mathematics—their sense of their discipline—in large measure from their experiences in classrooms. Students in the content courses for prospective teachers need to see teaching modeled in a manner that results in the kind of learning described in the MET recommendations. How and where do mathematics faculty with a content-specific knowledge base and, in many instances, little or no pedagogical expertise, develop this expertise? The MET recommendations point out that:

...mathematics faculty teaching courses for future teachers frequently have limited expertise about what mathematics to teach and how to teach it in a fashion that is appropriately connected to the school classroom situations these students will encounter (Ch. 1, p. 1).

As models of implementation for undergraduate content courses are developed, what mechanisms need to be in place to ensure that two-year college faculty also have opportunities to develop this expertise? Presently, little or no feedback is provided to many two-year college faculty about the effectiveness of their students’ general education content course preparation prior to transfer. These instructors frequently work in isolation and make choices unguided by a shared, clearly articulated set of goals and objectives. Will the choices in content and instructional strategies made in the mathematics content courses of the first two years contribute to the development of profound understanding or to the further fragmentation in students’ mathematical knowledge?

**Thinking Outside the Box**

As we explore ways to improve the undergraduate experiences of future teachers, we need to expand our domain of inquiry and look to other areas of expertise which could be very helpful. Research on student learning of mathematics offers us insights into how students learn and think about mathematics and provides us with frameworks in which to examine our beliefs, our values, and our instructional practices. A convergence of evidence indicates that, in order for long-term effective learning to occur, several things must happen:

- Newly-acquired skills and knowledge must be extended beyond the initial context.
- Mental representations must include knowledge of similarities and differences among problems.
- Students need to know when and how what is learned can be used.
- Students need to understand and apply underlying principles.

Universities whose primary mission includes education research have been asked to take on a decade-long intensive research initiative which includes “a comprehensive yet focused examination of how to improve teacher educating and teaching in mathematics, science, and technology,
as well as how to improve teacher retention in these subject areas.” The presidents and chancellors of 61 of the nation’s leading research universities already have committed their institutions to engaging in research that will enhance the practice of teaching. This national research initiative, together with the new national digital library which will have “primary responsibility for collecting, indexing, and broadly disseminating the results of existing and future research on the improvement of teacher education and teaching” can help focus our efforts in ways that will effectively improve the undergraduate experiences of future teachers (NRC, p.122).

**Conclusion**

Some of the issues involved in meeting the challenges of implementing the MET recommendations have been examined in this paper. One obvious conclusion is that without partnerships, articulation issues which are currently impediments to improving the undergraduate preparation of future teachers will not be resolved, and the development of consensus about what constitutes “deep understanding of school mathematics” and valid assessment(s) of the kinds and levels of understanding of mathematics students will remain an open question. The establishment of more essential partnerships with two-year colleges is critical. Identification and implementation of instructional experiences that build on and reinforce the understandings acquired in multi-course programs should be accessible to meet on-going professional development needs of classroom teachers and of the faculty who teach mathematics content courses for future teachers.

Several fundamental questions remain unanswered. They require careful examination and responses developed in an open, collaborative process. As we address the issues raised by the MET recommendations, we are challenged to expand our domain of inquiry and think “outside the box” in order to create and implement a coherent vision of mathematics teaching and learning—a vision shared by a broad community of mathematicians, mathematics educators, and mathematics education researchers from two-year and four-year colleges and from research universities.

**References**


