Acknowledgments

The moral is it hardly need be shown,
All those who try to go it sole alone,
Too proud to be beholden for relief,
Are absolutely sure to come to grief.

– Frost, Moral

The writing of this dissertation has been a challenging adventure—across seas, venturing forth to explore new vistas, making new discoveries, mathematical, visual, and musical, fulfilling long-held dreams, and at home—facing the herculean task of assembling the bits and pieces of new knowledge into the cognitive collage that is this thesis—asembled with the threads of intuition and analysis, shaped by the ambered heat of debates, and refreshed by quiet reflections down peaceful roads, alone and with colleagues and friends.

It has required a delicate juggling act—trying to balance my roles of student, observer, listener, and researcher, with those of wife, mother, teacher, friend and author. The debt of gratitude I owe to numerous friends, colleagues, and my family for their patience, support, humour and generosity is one I acknowledge but cannot hope to repay.

To my thesis advisor, Dr. David Tall—my sincere thanks and appreciation for a most remarkable four years of adventure and challenge and joy. His writings opened up the world of mathematics education research for me. His knowledge, expertise, humour, patience, and encouragement set standards of excellence and scholarship set me on journeys, intellectual and historical, which I shall long remember and which I shall try to live up to. His love of music and drama introduced me to the music of the night. May the challenges and adventures long continue!

I wish to thank Dr. Eddie Gray for his advice, insights, and critiques which proved invaluable—and inspired me to think about ways to present the data of this dissertation with greater clarity, which, it turned out, was the key to making sense of it after all.

To my colleagues and, in particular, my colleagues-in-arms, my textbook co-authors, Phil DeMaroios and Darlene Whitkanack, my heart-felt gratitude for their friendship and shared knowledge. They have nourished me intellectually, encouraged me professionally, and sustained me in moments of doubt.

To Keith Schwingendorf—my thanks for introducing me to the writings of David Tall, in the rain at Allerton, and during the remaining days of the conference.

I owe much gratitude to the students who have participated in this research study. They gave generously of their time and of themselves. They were wonderful teachers and supportive partners in this research.

Finally, I thank my husband, Bill, my sons, Bill, Mike, Tom, and John, and my extended family, including Jennifer, Laura, Debbie, and Molly, for their love, patience, and support. This thesis would never have been completed without their generosity and friendship. They are my compass—as I went forth on my adventures, constructing new cognitive collages of thoughts, caught up in the plethora of details, deadlines, and tasks yet-to-be-done—they were always there, patient witnesses to what really matters most in life—love, friendship, and shared visions.