CHAPTER 8  Visual Representations of Cognitive Collages:

The brain is just the wright of God,
For lift them, pound for pound,
And they will differ, if they do,
As syllable from sound.

– Emily Dickinson, The Brain is Wider than the Sky

8.1 A look back and an overview of what is yet to come

Two theses are the subject of this study. The first thesis—divergence and fragmentation of strategies occur between students of a undergraduate population of students who have demonstrated a lack of competence and/or failure in their previous mathematics courses—was investigated in Chapter 6 and the main research questions related to this thesis were addressed. Using their responses to pre- and post-test questions, the work of two groups of students, those most successful and those least successful, was described and interpreted within the theoretical framework outlined in Chapter 3. Evidence which support the thesis was presented. Chapter 7 continued the examination of this thesis, contrasting the responses and strategies of two students representative of each group of extremes. It was argued that the construction, organization, and reconstruction processes are constrained by a student’s initial perception(s) and the categorization of those perceptions, which cue selection and retrieval of a schema that directs subsequent actions and thoughts.

In this chapter the second thesis is examined—successful students construct, organize, and reconstruct their knowledge in ways that are qualitatively different from those of students who are least successful. Evidence drawn from analyses of student-constructed concept maps, triangulated within the framework of the cognitive collages of students in the extremes of a remedial undergraduate population will be presented in defence of this thesis. It is argued that, though students’ cognitive collages of knowledge representation structures are not directly knowable, it is possible to document the qualitatively different ways in which students construct and organize new knowledge, and restructure their existing cognitive structures, using student-constructed concept maps done at different points in time during the semester. Each map is a discrete representation at a particular moment in time, with maps on the same subject done at differ-
ent points in time during the semester. The longitudinal nature of the data collected by
this means provides a means by which the processes of construction, organization, and
restructuring that occurred are analyzed. When triangulated with the data already pre-
represented, the concept map analyses provide documentation of these processes of con-
struction.

The responses and strategies of two students, one from each group of extremes
were reported and analyzed in Chapter 7. The study of these two students continues in
this chapter, which begins with an examination of their concept maps, constructed dur-
ing week 4 and week 9 of the semester. The underlying structure of each map is
revealed and analyzed, using schematic diagrams of each map. Careful study of the
structure of the maps of MC (S2) and SK (S23), suggests radically different routes to
success and failure, routes which are also found in the maps of other students in each
group. An analysis of the development of classification schemas for all eight stu-
dents—the four most successful and the four least successful is presented. The chapter
concludes with a review of the evidence that supports the second thesis, based on the
data provided by students’ concept maps and the researcher’s schematic diagrams of
those maps. This evidence is triangulated and the classification schemes of the two
profiled students are examined within the broader context of the classification schemes
of the eight students of the two groups of extremes, those most and least successful.

Students were assigned concept maps on Function three times during the
semester; during weeks 4, 9 and 15. Each map was collected a week after its assign-
ment and retained as part of the data collection, though it was reviewed with the indi-
vidual student in order to clarify his/her intent and rationale for the connections
indicated on the map. Students did not have access to the completed maps during the
semester once these discussions had occurred. The maps of two students, MC (S2) and
SK (S23), are presented and analyzed. Their maps, like their work analyzed in the pre-
vious chapters, are typical of those created by the other students in their respective
groups. Even on first impression, the maps support the thesis that students organize
their knowledge in qualitatively different ways. More convincing evidence is provided
by the schematic diagrams which correspond to each student’s maps, which reveal the
underlying structure of the corresponding maps, and are indicative of the student’s
knowledge construction, organization, and reconstruction processes.
8.2 The Cognitive Collages of MC and SK

The concept maps of MC (Student 2), the student who started his college mathematics career in the self-paced arithmetic course in the Math Lab are examined first. The maps created during week 4 and week 9 respectively are shown in Figure 8.1 and 8.2 on the following page. His final concept map, created during week 15, was drawn on a very large piece of posterboard, which was not able to be scanned. Copies of his week 4 and week 9 concept maps, with the rough draft of his week 15 concept map are included in Appendix C. MC’s concept maps of Function are representations of his cognitive collages on Function at given moments in time. They convey, albeit imperfectly, the nature of knowledge construction that has occurred over time. MC has visualized his notion of Function in a way that reflects his unique way of thinking and organizing his knowledge about Function.

The words along the outer edges of the Week 4 central image of Function as a function machine is changing [left edge] and quantities [right edge] as you view the map. By week 9, MC appears to have enriched his concept images of representations and equations and added to his cognitive collage a new cognitive unit, finite differences. Though the shape of the central figure has been modified, his second map resembles the first, and the basic features of the first map are retained. Concept images of the notions measures of central tendency and measures of variability remain virtually unchanged from week 4. As these topics were used to introduce the notion of function and not revisited, it is not surprising that no new knowledge of these topics has been included on the week 9 map. The topics included on the maps by MC appear to follow the sequence of instruction, though the organization of those topics and the connections shown are uniquely his own. In his final interview, MC commented on the construction of his week 15 concept map:

While creating my [final] concept map on function, I was making strong connections in the area of representations. Specifically between algebraic models and the graphs they produce. I noticed how both can be used to determine the parameters, such as slope and the y-intercept. I also found a clear connection between the points on a graph and how they can be substituted into a general form to find a specific equation. Using the calculator to find an equation which best fits the graph is helpful in visualizing the connection between the two representations. I think it’s interesting how we learned to find finite differences and finite ratios early on and
then expanded on that knowledge to understand how to find appropriate algebraic models.

FIGURE 8.1. MC: Concept Map of Function Week 4

FIGURE 8.2. MC: Concept Map of Function Week 9
The maps in Figure 8.3 and Figure 8.4 were completed and submitted during week 4 and week 9 by SK.

**FIGURE 8.3.** SK: Concept Map of Function Week 4

![Function Week 4 Concept Map](image)

**FIGURE 8.4.** SK: Concept Map of Function Week 9

![Function Week 9 Concept Map](image)
The sparse maps of SK provide a sharp contrast to those of MC. There is no interiority to any of the concepts identified on SK’s maps of week 4 and of week 9. The week 4 map includes definitions, evidence of her belief discussed in the preceding chapter that she needs to know the definitions before she can learn about a concept. The week 9 map consists only of names of concepts, a bare skeleton of a cognitive collage, with no definitions included. One wonders what SK has associated with the labels she has included on her maps—what the nature of the connections she has indicated might be. Her final map [week 15] in Figure 8.5 contains fewer concepts branching from the main topic of the map, *Function*. The basic functions studied in the course, linear, quadratic, and exponential, together with procedures associated with each of these function types, have been incorporated under the main heading, *parameters* on her map.

**FIGURE 8.5. SK: Concept Map of Function Week 15**

This final concept map, completed in the week just prior to the final exam, provides additional information about what SK did or did not know about parameters on the day of the final exam. This question arose in the preceding chapter when her work on the rocket problem was analyzed. The concept map completed in week 15 suggests
that SK has constructed a concept image of *parameters* which consists of an association with the general form of each type of function; a recognition that the letters $a$, $b$, and $c$ are called *parameters*; and procedures by which the parameters are calculated for linear and exponential functions. The map does not include any procedure for calculating the parameters of a quadratic function. Given the fact that SK has included step-by-step procedures for determining the parameters of a linear function and of an exponential function, the absence of a procedure associated with quadratic functions, supports the conclusion that SK has no schema for determining the parameters of a quadratic function. Her final map once again includes definitions of terms like *simplify* (to get the lowest form); *evaluate* (when you’re looking for output); and *solve* (when you are looking for input). The lack of definitions on the week 9 concept map is perhaps, an indication that by week 9 SK had not yet clarified her understanding of these terms. The inclusion of definitions on the week 15 map reinforces the notion that she remains convinced of the importance of having a definition first. SK identified her strengths and weaknesses based on her final map. The procedural nature of her learning is confirmed during the interview:

The one operation I feel the strongest about is solving mathematical statements. When given an equation or an inequality I can solve for $X$. When solving an equation you have the output and are looking for the input. You must get one of the variables alone on one side of the equal sign. Once you have isolated one of the variables you must simplify to solve the problem. This type of solving is done algebraically. When evaluating a function you must know the input and use it to get the output. When given the input you need to substitute that into the equation to receive your answer.

Note SK’s use of the word *concept* when referring to these *processes* of solving an equation and evaluating a function:

These concepts are clearly labeled on my concept map.

It should be noted that her concept images of *solve* and of *evaluate* on the concept map are the reverse of what she said a week later in this interview. It was pointed out to SK during the interview that she had indicated a different interpretation of these two processes on her final map. Her surprise, when she subsequently examined her map, indicated that she was unaware she had formed two separate concept images for
these processes and that, at different times under different circumstances, she retrieved one or the other. Sk’s greatest weakness, which she described during her interview was:

My greatest weakness on my concept map is understanding linear, quadratic, and exponential functions. After using my notes and the book I was able to put together some of the pieces of my confusion. I can get the formulas but when I have a specific problem I don’t know which formula to choose to complete the problem.

8.2.1 Goals of Learning: MC and SK

One of MC’s goals of learning was to connect new knowledge to his prior knowledge and to build connections between and among concepts. Recall his interview comments at the end of the semester cited previously:

I have been able to utilize mathematical knowledge that I have gained from previous courses. It’s a good feeling to see things connecting together as I move further along in the text. As I go from investigation to investigation I really see connections in material that are clear and that help establish a solid body of knowledge. Another process that was very helpful in understanding algebra (specifically factoring) was using a graph to find the $x$-intercepts to find the zeros of an equation. This is a procedure I had never seen before, but I was able to connect it to my prior knowledge.

SK also expressed the desire to relate new knowledge to her prior learning. When describing her weaknesses as she perceived them based on constructing her final map, she said:

Also, when looking at a graph I can’t tell if it is linear, quadratic, or exponential. This was the first time I have worked with these functions and I think that may have been part of the problem because I had no prior knowledge to build on.

There are other areas that I feel that I have a strong understanding about as well. I choose mathematical statements because I was able to use my past knowledge and the new knowledge I have obtained this semester to have a better understanding.

Both students indicate they have the same overall goal of learning: the connection of new knowledge to prior knowledge. They are in the same classroom environment, both attend class regularly, and both work at learning mathematics, highly
motivated to succeed. In terms of competency, MC and SK appeared to have similar strengths and weaknesses at the beginning of the semester, based on their pre-test responses. They even had similar prior experiences in learning mathematics and their beliefs were conditioned by these experiences. Yet, even with a similar foundation, their performances diverged during the sixteen weeks, a divergence also reflected in their attitudes and beliefs. MC described his prior experiences and beliefs early in the semester in an interview:

When I started this course I had the misconception that all of the algebraic formulas would be given to us, and we would just have to follow a process to solve for them.

He recognized the need to take responsibility for his own learning. At mid-term, after receiving back an exam on which he was disappointed with his performance compared with a group exam done shortly before the individual exam, he said:

I've never been very good at taking math exams, I find that a lot of what I know slips away when it’s time to show and prove. To be honest, I wasn’t as prepared as I could have been. After receiving a score of nearly proficient on the test I took it upon myself to go back to Section 1.4 in the book and review everything from that point on in order to fully understand the material. I did that because I realize that math is a very progressive subject, and if I were to continue forward with minimal understanding of the previous sections, I would surely have minimal understanding of the rest of the sections.

SK, on the other hand, though she describes similar prior experiences and the beliefs that were shaped by those experiences, sees the responsibility for learning in a different light. In her first interview, SK described her beliefs about mathematics:

I thought math was about doing a lot of the same problems in order to have an understanding of what you were learning. I have always thought that doing mathematics meant doing operations, with formulas. I believe that learning math with concrete functions, definitions, and examples is the best way for me to learn math.

By the end of the semester, SK indicates very little change in her beliefs. She has shifted some of what she perceives to be the teacher’s role to that of her fellow students, who are members of her group:
After being in this class I realize that you can learn mathematics with less teacher-student interaction and more student-student interaction. I have started to take responsibility for my actions.

She shifts some of the responsibility for her failure to do well on (a) the text; (b) the graphing calculator; and (c) the pace of instruction.

- The most challenging thing is the fact that there is not direct formulas and direct “teaching.” I need an example which allows me to see how to do the work and then I could actually do the work for myself.

- A change that would help to improve my learning in this class would be a slower and more thoroughly explanation of the graphing calculator. I think more explanation on how everything ties together would be helpful. For example, a step by step process of why this part of the problem goes into the calculator and so on.

- I have a difficult time because so much depends on what I do. I have to keep up with all the assignments and I can’t let myself fall behind because if I miss one day of work I have no idea what is going on.

MC’s attitudes and beliefs have undergone a change during the semester, those of SK have been impacted to a far lesser extent. MC focuses on understanding why, SK focuses on understanding how to—clear-cut examples of the relational understanding and the instrumental learning described by Skemp [1987, pp. 166–172]. MC is willing and able to change his beliefs; SK holds fast to her previous beliefs, despite her growing frustration and lack of progress. In her final interview, she says, “Even now, I believe that learning math with concrete functions, definitions and examples is the best way for me to learn math.” Her early and later responses characterize this student and reflect the value she attaches to repetition and procedural rules which shape the construction and organization of her cognitive collages of cognitive units, concept images and schemas. MC, given the opportunity, chooses to travel the path towards proceptual understanding. SK, offered the same opportunities, stays her course on the path of divergence towards the proceptual divide.

8.3 Underlying Structure: Schematic Diagrams

Can the concept maps of these two students contribute more bits and pieces of knowledge that might shape the cognitive collages of MC and SK and inform our understanding of these two students? Schematic diagrams of their concept maps are
analyzed, together with the classification schemes each used when constructing their maps, in an effort to better understand the nature of their processes of constructing, organizing, and reconstructing their knowledge. The underlying structures of each map done by MC and SK during week 4, week 9, and week 15 are revealed in the schematic diagrams of those maps. They illustrate the differences in the nature of the processes of construction and reconstruction used by MC and SK. The concept maps of every student in each of the groups of extremes was analyzed in a similar fashion. The schematic diagrams of the maps of MC and SK are typical of the diagrams of the other students’ concept maps in their respective groups and are shown in Figure 8.6 (MC) and Figure 8.7 (SK) on the following two pages.

The schematic diagrams maintain a one-to-one correspondence with the named concepts, processes, and representations included on students’ original maps. Each node of the schematic diagram corresponds to one named concept, process or representation from the original map. The schematic diagrams were created using a background grid which imposed a degree of regularity on the relative positions of the various nodes and main branches. Other than this degree of regularity, the approximate location of each main branch [indicated by a slightly larger rectangle] relative to the central rectangle representing Function has been maintained; as has the approximate location of each node relative to its category as assigned by the student. Upon completion, the schematic diagrams were scaled so that all three schematic diagrams for a given student could be displayed on the same page to facilitate analysis.

The week 4 map elements are unpatterned. Those elements in the schematic diagrams of week 9 and week 15 that are unpatterned are the elements that were on a previous map [and diagram] which had been retained in the same relative position and within the same category. Beginning with the week 9 map, concepts, processes, and representations that are new [i.e. not included on a previous map/diagram] are indicated by gray-coloured nodes. A boldly-outlined and striped node represents an element that was present on an earlier map and is now in a different category and/or relative position. The maps are arranged from earliest (top) to latest (bottom), beginning at the top of the page with the schematic diagram of the week 4 concept map. Both the maps and their corresponding schematic diagrams illustrate the development over time of a student’s cognitive collage of the notion of function.