TIME VALUE OF MONEY

Key Terms and Concepts to Know

Present Value:
There is an old saying that time is money. Applied to accounting, it means that a dollar today is worth more to an investor or company than a dollar to be received in the future. The sooner the dollar is received, the longer it can be invested and used to generate more dollars. Therefore in order to properly compare a series of cash inflows and outflows occurring in various years, present value must be used to restate all of the cash inflows and outflows in current period dollars.

- Present value is based on compound interest, that is, current period interest is based on the principal amount plus the interest for all prior periods.
- Future cash flows are discounted back to the year when the bond was issued. The term discounting is appropriate because the future cash flows are worth less than their full amount today because we had to wait to receive them.
- Certain future cash flows may be annuities if they consist of equal amounts received of paid with equal frequency. Annuities are discounted using the Present value of an Annuity of $1 table.
- All other future cash flows are considered single payment cash flows. Single payments are discounted using the Present Value of $1 table.

The Value of a Dollar Depends on When The Dollar Is Received

- There are two aspects to the time value of money that are mirror images of each other. They differ only in the time period in which the unknown dollar amount occurs:
  - Future Value refers to a situation in which the present dollar amount and the interest rate are known and the unknown dollar amount occurs in a future time period.
  - Present Value refers to a situation in which the future dollar amount and the discount (interest) rate are known and the unknown dollar amount occurs in the present time period.
- A dollar received today is worth more than a dollar received sometime in the future. Since the dollar cannot be invested until it is received, the sooner it is received, the sooner it can be invested and earning a return and the more it will be worth at any time in the future.

- A dollar received in the future is worth less than a dollar received today. It is worth less today because the dollars earned from investing the dollars have not yet been earned.

The Timing of Cash Flows and Annuities

- The cash flows can be received (inflows) or paid out (outflows).
- The cash flows may occur:
  - In a single period
  - In the same amount in multiple periods
  - In different amounts in multiple periods
- Cash flows either received or paid in equal amounts and with equal frequency are an annuity.
  - If a child receives an allowance of $5 per week every Friday night; that is an annuity.
  - If a child receives an allowance of $5 per week whenever the parents remember to pay it; that is not an annuity.
  - If a child receives an allowance of a different amount depending on the chores completed that week every Friday night; that is not an annuity.

Time Value of Money Calculations

- Calculations can be performed using mathematical formulas, factors determined from mathematical tables or built-in functions in computer programs such as Excel.
- This module focuses on the use of the mathematical tables, popularly referred to as Future Value Tables and Present Value Tables.
- To make the calculations as simple as possible, there are separate present value tables for single cash flows and annuities and separate future value tables for single cash flows and annuities.
- **ALL FOUR TABLES LOOK VERY SIMILAR. BE SURE TO USE THE PROPER TABLE.**
- Compound interest and present value are mirror images of each other:
  - Compound interest assumes that the current investment (present value) and interest rate are known and the future value is to be calculated. The future value is calculated as follows: $FV = PV\ (1+i)^n$ where i is the interest rate and n is the number of periods.
  - Present value assumes that the future value(s) and discount rate are known and the present value is to be calculated. The present value is calculated as follows: $PV = FV / (1+i)^n$ where i is the interest rate and n is the number of periods.

  - In other words, multiply to solve for future value because future value will be larger and divide to solve for present value because present value will be smaller.
- Present value table converts $1 / (1+i)^n$ into a decimal to simplify the calculations.
- An annuity is a series of equal payments received or made with equal frequency. To eliminate the present value calculations for each payment in the series, the present value of an annuity table was developed. Using the distributive property [AB + AC = A(B+C)] the annuity tables sums the present value factors [B and C] for the number of periods to create an annuity factor which is multiplied by any one of the series of payments [A]. While this illustration has only two payments, the annuity table works for any number of periods and payments.
- The factor from the present value table will always be less than 1. The factor from the annuity table will be the same as the factor from the present value table for period 1 and will be greater than 1 for every period thereafter.

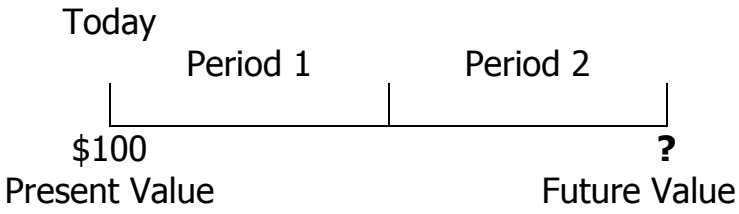
Solving Present Value and Future Value Problems

- There are a few simple rules to help determine the proper future value table to use:
  - Both future value tables will have factors greater than 1.000.
  - The future value of an annuity of $1 table will have factors larger than the future value of $1 table.
  - Always use the future value of $1 table unless the cash flows form an annuity.
  - Future values will always be greater than present values.
- There are a few simple rules to help determine the proper present value table to use:
  - The present value of $1 table will have factors less than 1.000.
  - The period 1 factors will be the same in both present value tables.
  - The present value of an annuity of $1 table will have factors greater than 1.000 beginning in period 2.
  - Present values will always be less than future values.
  - The typical accounting applications such as calculating bond proceeds and capital budgeting use only the present value tables.

Key Topics to Know

Future Value

- An interest rate of 12% (.12) per period is used in all examples.
- **The cash flow <u>single period</u> future value problem looks like this:**



Calculating the future value one period at a time:

Period 1:	$100	X	$(1 + .12)^{1}$	= $112.00
Period 2:	$112	X	$(1 + .12)^{1}$	= $125.44

Note that 1 is added to the interest rate to calculate the total amount at the end of the period (principal plus interest). The exponent is the number of periods.

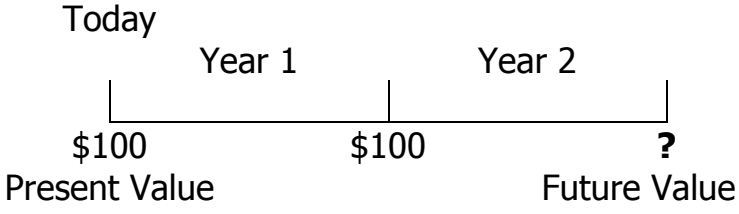
Calculating the future value in one step:

Years 1 and 2:	$100	X	$(1 + .12)^{2}$	= $125.44

Calculating the future value using the FV factor from the FV$1 table:

Years 1 and 2:	$100	X	1.2544	= $125.44

- **The cash flows of the <u>same amount in multiple periods</u> or <u>annuity</u> future value problem looks like this:**



Calculating the future value one year at a time:

Year 1:	$100	X	$(1 + .12)^{2}$	= $125.44
Year 2:	$100	X	$(1 + .12)^{1}$	= $112.00
Future value				$237.44

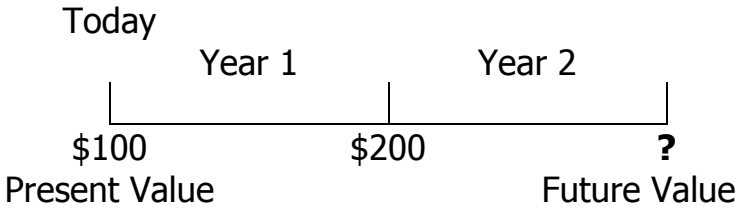
Calculating the future value one year at a time using the FV factor from the FV$1 table:

Year 1:	$100	X	1.2544	= $125.44
Year 2:	$100	X	1.1200	= $112.00
Future value			2.3744	$237.44

Calculating the future value using the FV factor from the FV of an annuity of $1 table:

Years 1 and 2:	$100	X	2.3744	= $237.44

- **The cash flows of different amounts in multiple periods future value problem looks like this:**



Because the cash flow amounts differ, their future values must be calculated separately.

Calculating the future value one year at a time:

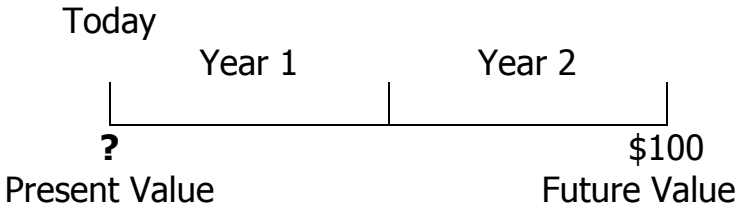
Year 1:	$100	X	$(1 + .12)^{2}$	= $125.44
Year 2:	$200	X	$(1 + .12)^{1}$	= $224.00
Future value				$349.44

Calculating the future value one year at a time using the FV factor from the FV$1 table:

Year 1:	$100	X	1.2544	= $125.44
Year 2:	$200	X	1.1200	= $224.00
Future value			2.3744	$349.44

Present Value

- An interest rate of 10% (.10) per period is used in all examples.
- **The cash flow <u>single period</u> present value problem looks like this:**



Calculating the present value one year at a time:

Year 2:	$100.00 /	$(1 + .10)^{1}$	= $90.91
Year 1:	$90.91 /	$(1 + .10)^{1}$	= $82.64

Note that 1 is added to the interest rate to calculate the total amount at the end of the period (principal plus interest). The exponent is the number of periods.

Calculating the present value in one step:

Years 1 and 2:	$100.00 /	$(1 + .10)^{2}$	= $82.64

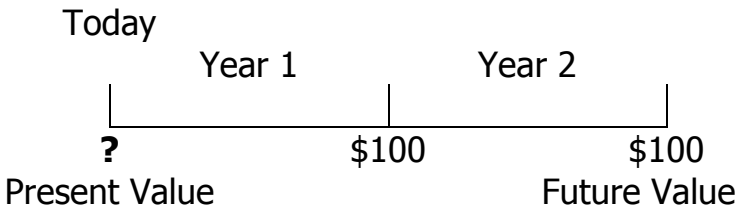
To simplify the calculations, the principal is multiplied by 1 divided by the future value factor (converting it to present value factor):

Years 1 and 2:	$100.00	X	$1/(1 + .10)^{2}$	= $82.64

Calculating the present value using the factor from the PV$1 table:

Years 1 and 2:	$100.00	X	.8264	= $82.64

- **The cash flows of the <u>same amount in multiple periods</u> or <u>annuity</u> present value problem looks like this:**



Calculating the present value one year at a time:

Year 2:	$100.00	/	$(1 + .10)^2$	= $82.64
Year 1:	$100.00	/	$(1 + .10)^1$	= $90.91
Present value				$173.55

To simplify the calculations, the principal is multiplied by 1 divided by the future value factor (converting it to present value factor):

Year 2:	$100.00	X	$1/(1 + .10)^2$	= $82.64
Year 1:	$100.00	X	$1/(1 + .10)^1$	= $90.91
Present value				$173.55

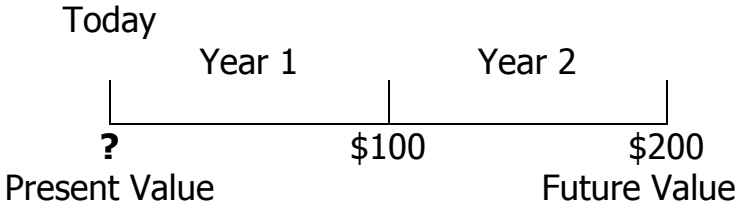
Calculating the present value one year at a time using the PV factor from the PV$1 table:

Year 2:	$100	X	.8264	= $82.64
Year 1:	$100	X	.9091	= $90.91
Present value			1.7255	$173.55

Calculating the present value using the PV factor from the PV of an annuity of $1 table:

Years 1 and 2:	$100.00	X	1.7355	= $173.55

- **The cash flows of <u>different amounts in multiple periods</u> present value problem looks like this:**



Because the cash flow amounts differ, their present values must be calculated separately.

Calculating the present value one year at a time:

Year 2:	$200.00	/	$(1 + .10)^2$	= $165.29
Year 1:	$100.00	/	$(1 + .10)^1$	= $ 90.91
Present value				$256.20

To simplify the calculations, the principal is multiplied by 1 divided by the future value factor (converting it to present value factor):

Year 2:	$200.00	X	$1/(1 + .10)^2$	= $165.29
Year 1:	$100.00	X	$1/(1 + .10)^1$	= $ 90.91
Present value				$256.20

Calculating the present value one year at a time using the PV factor from the PV$1 table:

Year 2:	$200	X	.8264	= $165.29
Year 1:	$100	X	.9091	= $ 90.91
Present value				$256.20

Example #1

ABC received a $10,000 inheritance on January 1. There were three options for receiving the cash: 1) in a single payment, 2) two payments of $5,000 each in years 1 and 3 or 3) five annual payments of $2,000 each. The payments will be invested at 6% interest. All payments will be received on the first day of the year.

Required:
a) Determine the future value of each payment option.
b) Which option provides the largest future value?

Solution #1

a)

	Cash Received	Period Received	Future Value Factor	Future Value
Option 1)	$10,000	1	1.3382	$13,382
Option 2)	$5,000	1	1.3382	$6,691
	$5,000	3	1.1910	5,955
	$10,000			$12,646
Option 3)	$2,000	1 - 5	5.9753	$11,951
	5 payments			
	$10,000			

b) In each option, a total of $10,000 will be received.
- Option 1) has the greatest future value because the all the cash is received on the earliest possible date and therefore will be invested for the greatest number of periods.
- Options 2) and 3) have a lesser future value as cash is in received later periods.

Note that Option 3) is the only annuity as equal payments ($2,000) are received with equal frequency (annually).

Example #2

ABC borrowed $50,000 from on January 1. ABC was offered three options for repaying the loan: 1) in a single payment at the end of 10 years, 2) two payments of $25,000 each in years 5 and 10 or 3) 10 annual payments of $5,000 each. ABC has other investments that earn 6% interest. All payments will be made on the last day of the year.

Required: a) Determine the present value of each repayment option.
b) Which option should ABC choose?

Solution #2

a)

	Cash Payment	Payment Period	Present Value Factor	Present Value
Option 1)	$50,000	10	.5584	$27,920
Option 2)	$25,000	5	.7473	$18,683
	$25,000	10	.5584	$13,960
	$50,000			$32,643
Option 3)	$5,000	1 - 5	7.3601	$36,801
	10 payments			
	$50,000			

b) In each option, a total of $50,000 will be received.

- Option 1) has the lowest present value because the all the cash is paid on the last possible date.
- Options 2) and 3) have increased future values as cash is repaid in earlier periods.

Note that Option 3) is the only annuity as equal payments ($5,000) are made with equal frequency (annually).

Accounting Problems and Present Value

Accounting problems frequently have a present value calculation for a combination of future lump sum payments and a series of periodic future payments. Rather than making the calculations, present value tables are used to simplify the calculations.

Present Value of a Lump Sum

- A lump sum is an amount expected to be received or paid in the future
- The unknown is what the amount is worth in today's dollars
- To solve, the following information must be known:
  - the number of interest compounding periods
  - the interest rate per compounding period
- Use the Present Value of $1 Table by selecting the row equal to the number of periods and the column equal to the interest rate

Present Value of an Annuity

- An annuity is a series of equal payments expected to be received or paid at regular future intervals
- The unknown is what the series of payments or receipts is worth in today's dollars?
- To solve, the following information must be known:
  - the number of interest compounding periods
  - the interest rate per compounding period
- Use the Present Value of an Annuity of $1 Table by selecting the row equal to the number of periods and the column equal to the interest rate

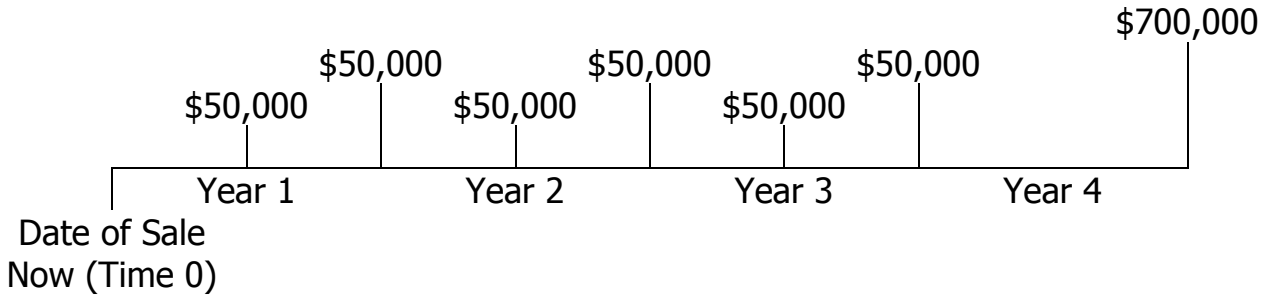
Example #3

D Company sold a building for $1,000,000. Payment terms were $50,000 to be paid semi-annually for years 1, 2 and 3 with the remaining balance to be paid at the end of year 4. The first payment will be made 6 months after the sale. The current rate of interest is 10%.

Required: Determine the present value of the future payments.

Solution #3

A picture of the cash payments:



Present value calculation:

- The lump sum payment factor is based on 5% interest (10% / 2 payments per year) and 8 periods (4 years x 2 payments per year).
- The annuity payment factor is based on 5% interest (10% / 2 payments per year) and 6 periods (3years x 2 payments per year).

	Cash Payment	Payment Period	Present Value Factor	Present Value
Lump-sum payment	$700,000	8	.6768	$473,760
Annuity payments	$50,000	6	5.0757	$253,785
				$727,545

Present Value and Future Value Tables

Present Value of 1

Periods	3%	4%	5%	6%	7%	8%	9%	10%	12%
3	0.9151	0.8890	0.8638	0.8396	0.8163	0.7938	0.7722	0.7513	0.7118
4	0.8885	0.8548	0.8227	0.7921	0.7629	0.7350	0.7084	0.6830	0.6355
5	0.8626	0.8219	0.7835	0.7473	0.7130	0.6806	0.6499	0.6209	0.5674
6	0.8375	0.7903	0.7462	0.7050	0.6663	0.6302	0.5963	0.5645	0.5066
7	0.8131	0.7599	0.7107	0.6651	0.6227	0.5835	0.5470	0.5132	0.4523
8	0.7894	0.7307	0.6768	0.6274	0.5820	0.5403	0.5019	0.4665	0.4039
9	0.7664	0.7026	0.6446	0.5919	0.5439	0.5002	0.4604	0.4241	0.3606
10	0.7441	0.6756	0.6139	0.5584	0.5083	0.4632	0.4224	0.3855	0.3220

Future Value of 1

Periods	3%	4%	5%	6%	7%	8%	9%	10%	12%
3	1.0927	1.1249	1.1576	1.1910	1.2250	1.2597	1.2950	1.3310	1.4049
4	1.1255	1.1699	1.2155	1.2625	1.3108	1.3605	1.4116	1.4641	1.5735
5	1.1593	1.2167	1.2763	1.3382	1.4026	1.4693	1.5386	1.6105	1.7623
6	1.1941	1.2653	1.3401	1.4185	1.5007	1.5869	1.6771	1.7716	1.9738
7	1.2299	1.3159	1.4071	1.5036	1.6058	1.7138	1.8280	1.9487	2.2107
8	1.2668	1.3686	1.4775	1.5938	1.7182	1.8509	1.9926	2.1436	2.4760
9	1.3048	1.4233	1.5513	1.6895	1.8385	1.9990	2.1719	2.3579	2.7731
10	1.3439	1.4802	1.6289	1.7908	1.9672	2.1589	2.3674	2.5937	3.1058

Present Value of an Annuity of 1

Periods	3%	4%	5%	6%	7%	8%	9%	10%	12%
3	2.8286	2.7751	2.7232	2.6730	2.6243	2.5771	2.5313	2.4869	2.4018
4	3.7171	3.6299	3.5460	3.4651	3.3872	3.3121	3.2397	3.1699	3.0373
5	4.5797	4.4518	4.3295	4.2124	4.1002	3.9927	3.8897	3.7908	3.6048
6	5.4172	5.2421	5.0757	4.9173	4.7665	4.6229	4.4859	4.3553	4.1114
7	6.2303	6.0021	5.7864	5.5824	5.3893	5.2064	5.0330	4.8684	4.5638
8	7.0197	6.7327	6.4632	6.2098	5.9713	5.7466	5.5348	5.3349	4.9676
9	7.7861	7.4353	7.1078	6.8017	6.5152	6.2469	5.9952	5.7950	5.3282
10	8.5302	8.1109	7.7217	7.3601	7.0236	6.7101	6.4177	6.1446	5.6502

Future Value of an Annuity of 1

Periods	3%	4%	5%	6%	7%	8%	9%	10%	12%
3	3.0909	3.1216	3.1525	3.1836	3.2149	3.2464	3.2781	3.3100	3.3744
4	4.1836	4.2465	4.3101	4.3746	4.4399	4.5061	4.5731	4.6410	4.7793
5	5.3091	5.4163	5.5256	5.6371	5.7507	5.8666	5.9847	6.1051	6.3528
6	6.4684	6.6330	6.8019	6.9753	7.1533	7.3359	7.5233	7.7156	8.1152
7	7.6625	7.8983	8.1420	8.3938	8.6540	8.9228	9.2004	9.4872	10.0890
8	8.8923	9.2142	9.5491	9.8975	10.2598	10.6366	11.0285	11.4359	12.2997
9	10.1591	10.5828	11.0266	11.4913	11.9780	12.4876	13.0210	13.5795	14.7757
10	11.4639	12.0061	12.5779	13.1808	13.8164	14.4866	15.1929	15.9374	17.5487

Practice Problems

Practice Problem #1

A company borrowed cash from the bank by signing a 5-year, 8% installment note. The present value of an annuity factor at 8% for 5 years is 3.9927. Each annual payment equals $75,000.

Required: Calculate the present value of the note.

Practice Problem #2

On January 1, a company issues 8%. 5 year, $300,000 bonds that pay interest semiannually. On the issue date, the annual market rate of interest is also 8%. The following information is taken from present value tables:

Required: Calculate the selling price of the bond.

Practice Problem #3

Explain the concept of the present value of a single amount.

Explain the concept of the present value of an annuity.

Practice Problem #4

Explain the difference between present value and future value.

Practice Problem #5

Compute the present value of the following single amounts to be received at the end of the specified period at the stated interest rate.

Amount	Interest Rate	Periods
$40,000	7%	5 years
$20,000	6%	8 years
$50,000	20%	5 years compounded semiannually

Practice Problem #6

Compute the present value of the following annuity amounts to be received for the specified period at the stated interest rate.

Amount	Interest Rate	Periods
$4,000	3%	5 years
$2,000	6%	5 years compounded semiannually
$5,000	9%	7 years

Practice Problem #7

a. Alpha Company is considering prepaying their rent for the next 4 years to avoid a price increase. Currently they pay $8,000 per year. Calculate the present value of the rent payments to determine what amount Alpha should pay today if current interest rates are 12% and interest is compounded annually.

b. Omega, Inc. won a lawsuit and will be receiving $400,000 at the end of 5 years. Calculate the present value of this award if interest is compounded semiannually and the current interest rate is 1) 18% and 2) 10%.

c. Bonnie has just received news of an inheritance. She will be receiving $10,000 per year for the next 20 years and a lump sum payout after 20 years of $200,000. Calculate the present value of her inheritance if the current interest rate is 9% and it is compounded annually.

True / False Questions

1. An interest rate is also called a discount rate.
   True False

2. The number of periods in a present value calculation may only be expressed in years.
   True False

3. The present value factor for determining the present value of $6,300 to be received three years from today at 10% interest compounded semiannually is 0.7462.
   True False

4. The present value of $1 formula is often useful when a loan must be repaid in full at a later date and the borrower wants to know how much to invest today to be able to make the future payment.
   True False

5. In a present value or future value table, the length of one time period may be interpreted as one year, one month, or any other length of time.
   True False

6. The present value of $2,000 to be received nine years from today at 8% interest compounded annually is $1,000.
   True False

7. Future value can be found if the interest rate, the number of periods, and the present value are known.
   True False

8. An annuity is a series of equal payments occurring at equal intervals.
   True False

Multiple Choice Questions

1. To determine the value of present-day assets at a future date, the computations that should be used are:
   a) Present value
   b) Annuity
   c) Interest
   d) Future value

2. Which interest rate column would be used from a present value or future value table for 8% interest compounded quarterly?
   a) 16%
   b) 8%
   c) 4%
   d) 2%

3. A loan that requires a single payment of $4,000 at the end of 3 years. The loan's interest rate is 6%, compounded semiannually. How much was borrowed?
   a) $3,350.00
   b) $3,358.40
   c) $4,000.00
   d) $3,660.40

4. ABC received an inheritance and wants to invest a sum of money today that will yield $5,000 at the end of each of the next 10 years. Assuming an interest rate of 5% compounded annually, how much must be invested today?
   a) $50,000.00
   b) $47,500.00
   c) $45,125.00
   d) $38,608.50

5. A company expects to invest $5,000 today at 12% annual interest and plans to receive $15,529 at the end of the investment period. How many years will elapse before the company accumulates the $15,529?
   a) 10 years
   b) 3.1 years
   c) 5 years
   d) 8 years

6. What interest rate is required to accumulate $6,802.50 in four years from an investment of $5,000?
   a) 5%
   b) 8%
   c) 10%
   d) 12%

7. Russell Company has acquired a building with a loan that requires payments of $20,000 every six months for 5 years. The annual interest rate on the loan is 12%. What is the present value of the building?
   a) $72,096
   b) $113,004
   c) $147,202
   d) $86,590

8. A company needs to have $150,000 in 5 years, and will create a fund to insure that the $150,000 will be available. If it can earn a 6% return compounded annually, how much must the company invest in the fund today to equal the $150,000 at the end of 5 years?
   a) $141,000
   b) $112,095
   c) $100,000
   d) $45,000

Solutions to Practice Problems

Practice Problem #1

Present value of principal	$75,000 x 3.9927 =	$299,453

Practice Problem #2

Interest paid each period	$300,000 x 8% / 2 =	$12,000
Present value of principal	$300,000 x .6756 =	$202,680
Present value of interest	$12,000 x 8.1109 =	97,330
		$300,010

Note that the $10 difference is due to rounding of the present value factors.

Practice Problem #3

The present value of a single amount is used to find today's value for an amount to be received at a future date. It is equal to the amount that can be invested now at the specified interest rate to yield the future value.

The present value of an annuity is the amount that can be invested now at the specified interest rate to yield a future series of equal periodic payments.

Practice Problem #4

Present value tells us the value today of receiving some amount in the future. Future value is the value that an amount today will grow to in the future. The difference between the present value and the future value is the time value of money.

Practice Problem #5

Amount		Present Value
$40,000	40,000 x .7130 =	$28,520
$20,000	$20,000 x .6274 =	$12,548
$50,000	$50,000 x .3855 =	$19,275

Practice Problem #6

Amount		Present Value
$4,000	$4,000 x 4.5797 =	$18,319
$2,000	$2,000 x 8.5302 =	$17,060
$5,000	$5,000 X 5.0330 =	$25,165

Practice Problem #7

a)

	Annuity Factor 4 periods at 12%	
$8,000 x	3.03735 =	$24,299

b)

	Present Value of $1 10 periods at 5%	
$400,000 x	.42241 =	$168,964
	Present Value of $1 10 periods at 9%	
$400,000 x	.61391 =	$245,564

c)

	Annuity Factor 20 periods at 9%	
$10,000 x	9.12855 =	$91,286
	Present Value of $1 20 periods at 9%	
$200,000 x	.17843 =	$35,686
		$126,972

Solutions to True / False Problems

1. True
2. False – the number of periods may be expressed in days, weeks, months, years or any other time period.
3. True
4. False – since the unknown value is at a future date, the future value tables must be used.
5. True
6. True
7. True
8. True

Solutions to Multiple Choice Questions

1. D
2. D
3. A
4. D
5. A
6. B
7. C
8. B